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Q1. a)

$r > R$

$E = \frac{Q_{enc}}{4\pi r^2 \varepsilon_0} = \frac{Q}{4\pi r^2 \varepsilon_0}$

$Q_{enc} = Q$

$r \leq R$

$E = \frac{Q_{enc}}{4\pi r^2 \varepsilon_0} = \frac{rQ}{4\pi R^3 \varepsilon_0}$

$Q_{enc} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} Q$

b) $dF = \frac{q \cdot dV \cdot Q}{4\pi r^2 \varepsilon_0 V}$ ?

c) $F = \frac{q \cdot Q}{2} \int_0^R \frac{4\pi r^2 \, dr}{4\pi r^2 \varepsilon_0 \cdot 4\pi R^3} = \frac{qQ}{2\varepsilon_0 R^2}$ ?

Q2.

a) $\sigma = \frac{Q}{A}$

$E = \frac{\sigma}{\varepsilon_0} \hat{n} = \frac{q_0 \sin(\omega t)}{\pi a^2 \varepsilon_0} \hat{z}$ ✓

b) $\bar{\nabla} \times E = -\frac{\partial E_z}{\partial r} \hat{\phi} = 0 \Rightarrow \oint H \, dl = \frac{\partial}{\partial t} \int D \, ds \Rightarrow B = \frac{\mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \cdot \pi r^2}{2\pi r}$

$B = \frac{\mu_0 q_0 \cos(\omega t) \cdot \omega r}{2\pi a^2} \hat{\phi}$

c) $S = \frac{(E \times B)}{\mu_0} \Rightarrow \frac{E_z \cdot B_\phi}{\mu_0}(-\hat{r}) = \left(\frac{q_0 \sin(\omega t)}{\pi a^2 \varepsilon_0}\right) \cdot \left(\frac{q_0 \cos(\omega t) \cdot \omega r}{2\pi a^2}\right)(-\hat{r})$ ✓

$$E \times B = \begin{vmatrix} \hat{\rho} & \hat{\phi} & \hat{z} \\ E_\rho & E_\phi & E_z \\ B_\rho & B_\phi & B_z \end{vmatrix} = -E_z B_\phi \hat{\rho}$$

$$S = \frac{q_0^2 \omega r \sin(\omega t)\cos(\omega t)}{2\pi^2 a^4 \varepsilon_0}(-\hat{r}) = \frac{q_0^2 \omega r \sin(2\omega t)}{(2\pi a)^2 \varepsilon_0}$$ ✓

d) $\frac{\omega a}{c} \ll 1$ $\quad u = \frac{1}{2}\left(\varepsilon_0 \frac{q_0^2 \sin^2(\omega t)}{(\pi a^2 \varepsilon_0)^2} + \mu_0 \frac{q_0^2 \omega^2 r^2 \cos^2(\omega t)}{(2\pi a^2)^2 \mu_0}\right) = \frac{q_0^2 \mu_0}{2\pi^2 a^4}\left[c^2 \sin^2(\omega t) + \frac{\omega^2 r^2 \cos^2(\omega t)}{4}\right]$

$$\frac{\partial u}{\partial t} = \frac{q_0^2 \mu_0}{2\pi^2 a^4}\left[c^2 \cdot 2\underbrace{\sin(\omega t)\cdot\cos(\omega t)}_{\frac{1}{2}\sin(2\omega t)} \cdot \omega + \underbrace{\frac{\omega^2 r^2}{4} \cdot 2\cos(\omega t)\cdot(-\sin(\omega t)\cdot\omega)}_{}\right]$$

$$\nabla S = \frac{1}{r}\frac{\partial(r S_\rho)}{\partial r} = \frac{1}{r} \cdot \frac{q_0^2 \omega r \sin(\omega t)\cos(\omega t)}{\pi^2 a^4 \varepsilon_0} = \frac{q_0^2 \omega \sin(2\omega t)}{2\pi^2 a^4 \varepsilon_0}$$

$$\nabla S + \frac{\partial u}{\partial t} = 0 \Rightarrow \frac{q_0^2 \omega \sin(2\omega t)}{2\pi^2 a^4 \varepsilon_0} + \frac{q_0^2 \mu_0 \sin(2\omega t)\omega}{2\pi^2 a^4}\left[c^2 - \frac{\omega^2 r^2}{4}\right]$$

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$$\frac{q_0^2 \omega \sin(2\omega t)}{2\pi^2 c^4 \varepsilon_0} + \frac{q_0 \omega \sin(2\omega t)}{2\pi a^4 \varepsilon_0} \left[ 1 - \left(\frac{\omega r}{4c}\right)^2 \right] = 0$$ ✓

## Q3.

a) $|\langle \psi | \psi \rangle|^2 = 1 \Rightarrow \int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1 \Rightarrow \int_{-\frac{L}{2}}^{\frac{L}{2}} A^2 \cos^2\left(\frac{3\pi x}{L}\right) dx = 1$

$$A^2 \left[ \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{2} + \frac{\cos}{2}\left(\frac{6\pi x}{L}\right) dx \right] = 1 \Rightarrow A^2 \cdot \frac{L}{2} = 1 \Rightarrow \boxed{A = \sqrt{\frac{2}{L}}}$$

b) $\int_{-\frac{L}{4}}^{\frac{L}{4}} \frac{2}{L} \cdot \left( \frac{1}{2} + \frac{\cos}{2}\left(\frac{6\pi x}{L}\right) \right) dx = \frac{1}{2} + \frac{2}{L}\left( \frac{L}{12\pi} \sin\left(\frac{6\pi x}{L}\right) \right)\Big|_{-\frac{L}{4}}^{\frac{L}{4}}$

$$= \frac{1}{2} + \frac{1}{6\pi}\left[ \sin\left(\frac{3\pi}{2}\right) - \sin\left(-\frac{3\pi}{2}\right) \right] = \frac{1}{2} - \frac{1}{3\pi}$$

c) $\frac{\partial \psi}{\partial x} = A \frac{3\pi}{L} \left(-\sin\left(\frac{3\pi x}{L}\right)\right)$

$\frac{\partial^2 \psi}{\partial x^2} = -A \left(\frac{3\pi}{L}\right)^2 \cos\left(\frac{3\pi x}{L}\right) = -\left(\frac{3\pi}{L}\right)^2 \psi(x)$

$$-\frac{\hbar^2}{2m}\frac{\partial \psi}{\partial x^2} = E\,\psi(x) \Rightarrow \boxed{\frac{\hbar^2}{2m}\left(\frac{3\pi}{L^2}\right)^2 = E}$$

d) $k^2 = \frac{2mE}{\hbar^2} \Rightarrow k = \frac{n\pi}{L} \Rightarrow E = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$

$E_0 = \frac{\hbar^2\pi^2}{2mL^2} \Rightarrow \Delta E = E_3 - E_0 = \left(\frac{\hbar\pi}{L}\right)^2 \frac{1}{2m}\left[9 - 1\right] = \frac{4\hbar^2\pi^2}{mL}$

$\hbar = \frac{h}{2\pi}$

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = hc \cdot \frac{mL}{4\pi^2\hbar^2} \Rightarrow \boxed{\lambda = \frac{mLc}{2\pi\hbar}}$$

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Q4. $\frac{dN(t)}{dt} = -RN(t)$

a)

$$\frac{dN(t)}{N(t)} = -R\,dt \Rightarrow \boxed{N(t) = N(0)\cdot e^{-Rt}}$$

$$\frac{N(\bar{t})}{N(0)} = \frac{1}{e} = e^{-2\cdot 10^{-6}R} \Rightarrow 0 = -2\cdot 10^{-6}R + 1$$

$$\boxed{R = \frac{1}{2}\cdot 10^{6}} \left[\frac{1}{s}\right]$$

valor no referencial do muon

c) No referencial da Terra:

$\bar{t}' = \gamma\,\bar{t} = 35\cdot 10^{-6}\,s$

$d = \bar{t}'\cdot v_m = 35\cdot 10^{-6}\cdot 0{,}998c \simeq 9000\,m$

$\therefore N(\bar{t}) = \frac{10^8}{e}$

b) $d = \bar{t}\cdot v_m = 2\cdot 10^{-6}\cdot 0{,}998c = 600\,m$

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Q5.

a) $E = \frac{p^2}{2m} + mgh$

$Z = \int d^3p\, d^3x\, e^{-\beta\left(\frac{p^2}{2m} + mgh\right)} = \int_0^L d^3x\, e^{-\beta mgh} \cdot \int_{-\infty}^{\infty} d^3p\, e^{-\frac{\beta p^2}{2m}}$

$\alpha = \frac{\beta}{2m}$

$\left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} \Rightarrow \left(\frac{2m\pi}{\beta}\right)^{\frac{1}{2}}$

$$Z = \left(\frac{2m\pi}{\beta}\right)^{\frac{3}{2}} \cdot \left(\frac{e^{-\beta mgL} - 1}{-\beta mg}\right)$$

+ details

b)

$Z = \iiint_0^L \iiint_{-\infty}^{\infty} e^{-\beta\left(\frac{p_x^2 + p_y^2 + p_z^2}{2m}\right) - \beta mgz}\, dp_x dp_y dp_z\, dx dy dz$

$Z = \iint_0^L \iiint_{-\infty}^{\infty} e^{-\beta\left(\frac{p_x^2 + p_y^2 + p_z^2}{2m}\right)} dp_x dp_y dp_z \int_0^L e^{-\beta mgz}\, dz$

$L^2 \qquad \left(\frac{2m\pi}{\beta}\right)^{3/2}$

$$Z = \left(\frac{2m\pi L^2}{\beta}\right)^{\frac{3}{2}} \cdot \left[\frac{1 - e^{-\beta mgL}}{\beta mgL}\right]$$ ✓

b) $\langle u \rangle = -\frac{\partial \ln Z_u}{\partial \beta} = \frac{3}{2}\frac{\partial}{\partial \beta}\ln\beta = \frac{3}{2}kT$

$\ln Z_u = \frac{3}{2}\left[\ln(2m\pi L^2) - \ln\beta\right]$

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c) $\langle T \rangle = -\frac{\partial \ln Z_P}{\partial \beta} = \frac{-e^{-\beta mgL} \cdot mgL}{1-e^{-\beta mgL}} + \frac{1}{\beta}$

$\ln Z_P = +\ln\left(1-e^{-\beta mgL}\right) - \ln\left(\beta mgL\right)$

$= \frac{mgL}{1-e^{\beta mgL}} + \frac{1}{\beta}$

d) $e^{\beta mgL} \approx 1 + \beta mgL + (\beta mgL)^2 \ldots$

$= \frac{mgL}{1-1-\beta mgL} + \frac{1}{\beta} = 0$

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HS4

$I = \int r^2 dm = \int_{-\frac{D}{2}}^{\frac{D}{2}} r^2 \cdot \rho \cdot dr = \rho \cdot \frac{r^3}{3} \Big|_{-\frac{D}{2}}^{\frac{D}{2}} = \frac{\rho D^3}{12} = \frac{D^2 M}{12}$

Q6.

a) $\tau = I \cdot \alpha = \frac{mD^2}{4} \cdot$ (acc. ang.) $\quad \omega^2 = \frac{v^2}{r^2} \quad dm = dr\rho$

$\alpha = \frac{d\omega}{dt} \quad \Downarrow \quad F = ma = \frac{mv^2}{r}$ density linear

$\tau = dF = \frac{d \cdot m\omega^2 Dr}{2}$

**Q8.** $|\Psi(0)\rangle = \sum_n c_n |\phi_n\rangle$

a) $\sum_n |c_n|^2 = 1$

$|\Psi(t)\rangle = \sum_n c_n |\phi_n\rangle \cdot e^{-i\frac{E_n}{\hbar}t}$

Prob $n \geq 1$ =

$= \sum_{n=2}^{\infty} |c_n|^2 - |c_0|^2 - |c_1|^2$

b) $|\Psi(0)\rangle = c_0|\phi_0\rangle + c_1|\phi_1\rangle$

$\langle H\rangle = \langle\Psi(0)|\hat{H}|\Psi(0)\rangle = (\langle\phi_0|c_0 + \langle\phi_1|c_1)\,\hat{H}\,(c_0|\phi_0\rangle + c_1|\phi_1\rangle)$

$= E_0 c_0^2 + E_1 c_1^2$ $\quad E_0 = \frac{\hbar\omega}{2},\ E_1 = \frac{3}{2}\hbar\omega \quad E_0 c_0|\phi_0\rangle + E_1 c_1|\phi_1\rangle$

$\langle H\rangle = \hbar\omega \Rightarrow E_0|c_0|^2 + E_1|c_1|^2 = \hbar\omega \Rightarrow \frac{|c_0|^2}{2} + \frac{3}{2}|c_1|^2 = 1$

$|c_0|^2 + |c_1|^2 = 1$

$|c_0|^2 = 1 - |c_1|^2$

$\frac{3|c_1|^2}{2} = 1 - \frac{(1 - |c_1|^2)}{2}$

$3|c_1|^2 = 2 - 1 - |c_1|^2 \Rightarrow |c_1|^2 = \frac{1}{4}$

$|c_0|^2 = \frac{3}{4}$

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$\omega r$

**Q7.** a) $T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}m[\dot{r}^2 + r^2\omega^2] = L$

$x = r\cos(\omega t) \Rightarrow \dot{x} = \dot{r}\cos(\omega t) + r\sin(\omega t)\cdot\omega$

$y = r\sin(\omega t)$

b) $L \Rightarrow m\ddot{r} - mr\omega^2 = 0$

$\ddot{r} = r\omega^2$

c) $r = e^{\pm\omega t}$ $(r_0?)$

$p_k = \frac{\partial L}{\partial \dot{q}_k}$

$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}$

$\dot{r} = \frac{p_r}{m}$

d) $H = \sum p_k \dot{q}_k - L$

$H = m\dot{r}^2 - \frac{1}{2}m\dot{r}^2 - r^2\omega^2 = \frac{1}{2}m\dot{r}^2 - r^2\omega^2$

$H = \frac{1}{2}\frac{p_r^2}{m} - r^2\omega^2;$ $\quad \dot{r} = \frac{\partial H}{\partial p_r} = \frac{p_r}{m} \quad \dot{p}_r = -\frac{\partial H}{\partial r} = 2r\omega$

e) $\frac{\partial L}{\partial t} = 0$ energy conserved

$\frac{\partial H}{\partial t} = 0$ [illegible]

**c)** $\langle\Psi(0)|\hat{X}|\Psi(0)\rangle = (\langle\phi_0|c_0 + \langle\phi_1|c_1^*)\hat{X}(c_0|\phi_0\rangle + c_1|\phi_1\rangle) = \langle\hat{X}\rangle$

$c_0$ = real e posit.

$c_1 = |c_1| e^{i\theta_1}$

$$(\langle\phi_0|c_0 + \langle\phi_1|c_1^*) \cdot \left[\left(c_0\sqrt{\frac{\hbar}{2m\omega}}\sqrt{0+1}\,|\phi_1\rangle\right) + c_1\sqrt{\frac{\hbar}{2m\omega}}\sqrt{1}\,|\phi_0\rangle\right)\right]$$

$$c_0 c_1 \sqrt{\frac{1\hbar}{2m\omega}} + c_0 c_1^* \sqrt{\frac{\hbar(0+1)}{2m\omega}} = \frac{1}{2}\sqrt{\frac{\hbar}{m\omega}}$$

$$c_0 \cdot |c_1| e^{i\theta_1}\sqrt{\frac{1}{2}} + c_0|c_1| e^{-i\theta_1}\sqrt{\frac{0+1}{2}} = \frac{1}{2}$$

$|c_0| = \frac{\sqrt{3}}{2}$ $\quad |c_1| = \frac{1}{2}$

$c_0 \cdot c_1 = \frac{\sqrt{3}}{4}$

$$\sqrt{\frac{1}{2}}\, e^{i\theta_1} + e^{-i\theta_1}\sqrt{\frac{0+1}{2}} = \frac{1}{c_0|c_1|2} \Rightarrow e^{i\theta_1} + e^{-i\theta_1} = \frac{\sqrt{2}}{2c_0|c_1|}$$

$$2\cos(\theta_1) = \sqrt{\frac{2}{3}} \Leftrightarrow \theta_1 = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right) \quad 35°$$

**d)** $|\Psi(0)\rangle = \frac{\sqrt{3}}{2}|\phi_0\rangle + \frac{1}{2}\cdot e^{i\theta_1(0)}|\phi_1\rangle$

$$|\Psi(t)\rangle = \frac{\sqrt{3}}{2}\cdot e^{i\frac{\omega t}{2}}|\phi_0\rangle + \frac{1}{2}e^{i(\omega\frac{t}{2}+\theta_1(t))}|\phi_1\rangle$$

$\langle\hat{X}\rangle(t) \Rightarrow \langle\Psi(t)|\hat{X}(t)|\Psi(t)\rangle =$

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Q9.

a) $[L_i, L_j] = i\hbar \, \varepsilon_k \, \varepsilon_{ijk} L_k$

$\varepsilon_{ijk} = 1$ or $-1$ if cyclical index or 0 if repeated index

$$\vec{L} \times \vec{L} = \begin{vmatrix} i & j & k \\ L_i & L_j & L_k \\ L_i & L_j & L_k \end{vmatrix} = (L_j L_k - L_k L_j)\hat{i} + (L_k L_i - L_i L_k)\hat{j} + (L_i L_j - L_j L_i)\hat{k}$$

$$[L_j, L_k]\hat{i} + [L_k, L_i]\hat{j} + [L_i, L_j]\hat{k}$$

$$= i\hbar \left[ \sum_i^2 \varepsilon_{ijk} L_i + \sum_j^2 \varepsilon_{ijk} L_j + \sum_k^2 \varepsilon_{ijk} L_k \right] = i\hbar [L_i + L_j + L_k] = i\hbar \vec{L}$$

$\varepsilon_{ijk} = 1$, $\varepsilon_{ijk} = 0$

b) $\vec{L} = R \times P$ , $[R_i, P_j] = i\hbar \, \delta_{ij}$

$$[L_i, R_j] = L_i R_j - R_j L_i = (R_i \times P_i) R_j - R_j (R_i \times P_i)$$

$$L = R \times P = [R_j, P_k]\hat{i} + [R_k, P_i]\hat{j} + [R_i, P_j]\hat{k}$$

$$L = i\hbar \left[ \delta_{jk}\hat{i} + \delta_{ki}\hat{j} + \delta_{ij}\hat{k} \right]$$

Q6.

a) $P = \frac{aT^4}{3}$ , $U = aT^4 V$ , $T_0 V_0$

$\Delta U = aT_0^4 V_0$

$dU = dQ - dW$

$\Delta U = Q - W$

$W = P\Delta V$

$$\boxed{W = \frac{aT_0^4}{3} \cdot V_0}$$ ver[illegible].

$$\left(\frac{\partial U}{\partial T}\right)_V = 4aT^3 V$$

$$\left(\frac{\partial U}{\partial V}\right)_T = aT^4$$

b) $Q = aT_0^4 V_0 + \frac{aT_0^4}{3} V_0 = \boxed{\frac{4}{3} T_0^4 V_0}$

c) $0 = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] dV$

$0 = 4aT^3 V\, dT + \left[aT^4 + \frac{aT^4}{3}\right] dV \Rightarrow -4aT^3 V\, dT = \frac{4}{3} aT^4\, dV$

$V_0 T_0^3 = V_f T_f^3$ ; $T_f = \frac{T_0^3}{2}$

$\frac{-3dT}{T} = \frac{dV}{V} \Rightarrow -3\ln T = \ln V$

$$\boxed{T^{-3} = V}$$

$V = \frac{1}{T^3} = VT^3 = 1$

d) $W = -\Delta U = -a\Delta T^4 \Delta V = a(T_f^4 - T_0^4) \cdot V_0$

$W = a\left(\frac{T_0^{12}}{16} - T_0^4\right) V_0 = \boxed{\frac{aV_0 T_0^{12}}{16} - aT_0}$

(10.)